The crack resistance in polybutylene terephthalate (PBT) at crack initiation and during steady crack growth

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The resistance to crack initiation and quasi-static crack propagation is investigated for a polybutylene terephthalate (PBT) using annealed and unannealed three-point bend specimens. The resistance to crack initiation (R_i) is determined based on the generalized locus method which determines the resistance to crack growth including crack initiation utilizing the locus of characteristic points on the load against load-point displacement curves of specimens which differ only in initial crack length. This generalized locus method also enables us to investigate the invariance of the crack resistance value along the locus line. The steady state crack resistance (R_p) during quasi-static crack propagation is determined utilizing the functional relation between total essential energy (U_f) for complete fracture and the initial ligament length. The total essential energy is the sum of the blunting energy and the integration of the resistance to crack propagation with respect to the cracking area. The invariance of the crack initiation resistance R_i and the steady state resistance R_p is discussed based on the experimental results of the annealed and unannealed specimens which show different sizes of crack tip plastic deformation.

1. Introduction

Much research has been done in an effort to find fracture characterizing material properties for engineering materials. In linear elastic fracture mechanics (LEFM), the region of inelastic deformation at the crack tip is assumed to be small compared to the characteristic dimensions of the cracked body. The fracture process satisfying this condition can be characterized by parameters such as the stress intensity factor K or the strain energy release rate G.

However, when the inelastic deformation region is not small, as in ductile materials, the concept of elastic plastic fracture mechanics (EPFM) is necessary to describe the fracture process. One method, the J-integral [1], has been found useful in describing crack initiation and a certain amount of crack growth provided J-control and J-dominance requirements [2] are met. For crack growth beyond the useful range of the J-integral, the crack resistance R can be represented in terms of the total energy release rate $\tilde{G}$ [3] which is equivalent to the energy consumption rate in the vicinity of the crack tip during crack propagation.

Recently, we have reported [4] that both the crack initiation resistance and the resistance during crack growth can be determined utilizing the locus of characteristic points on the load *vs* load-point displacement curves of specimens which differ only in initial crack length. In this paper, the crack initiation resistance of PBT is investigated for both annealed and unannealed specimens based on this generalized locus method, and the invariance of the crack resistance value along the locus line is discussed. The total essential energy for complete fracture (U_f) which is the sum of the blunting energy and the integration of the resistance to crack propagation with respect to the cracking area is analysed based on an R-curve [5–7] model which consists of three distinct regions. The steady state crack resistance (R_p) during quasi-static crack propagation is determined based on this analysis.

2. Determination of the crack initiation resistance (R_i)

Previously, we have reported [8, 9] that the crack initiation resistance can be determined in terms of the critical J-integral value (J_c) utilizing the locus line of crack initiation points on load-displacement records. Recently, we have also reported [4] that the resistance to crack propagation, including the resistance at crack initiation, can be determined in terms of the total energy release rate $\tilde{G}$ from

$$\tilde{G} = -\frac{1}{B}\frac{dU_L}{da} \tag{1}$$

where B is the specimen thickness, a is the initial crack length, U_L is the area enclosed by a loading curve, a locus line of a set of characteristic points, and the displacement axis. Hence, by taking the crack initiation points as characteristic points, the crack initiation resistance R_i (b), which is equivalent to J_c at crack initiation for a given thickness, can be determined from

$$R_i\,(b) = \frac{1}{B}\frac{dU_{Li}}{db} \tag{2}$$

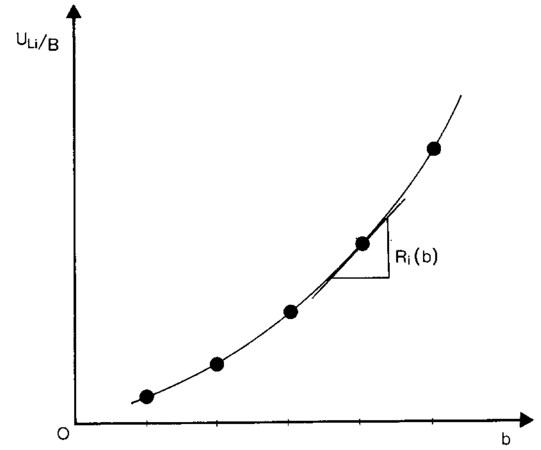

Figure 1 A schematic plot of U_{Li}/B against initial ligament length b. The slope is the crack initiation resistance R_i (b).

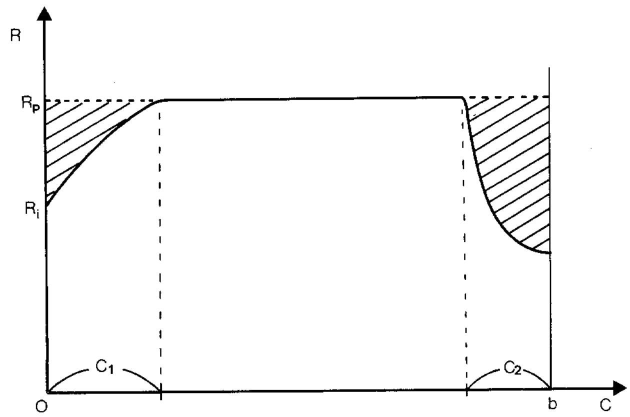

Figure 2 A schematic R-curve showing ascending, plateau, and descending regions where c is crack extension, and b is the initial ligament length.

where U_{Li} is U_L for crack initiation points, and $b = W - a$ where W is the specimen width. Shown in Fig. 1 is a schematic plot of U_{Li}/B with respect to ligament length b. The slope is the crack initiation R_i as a function of b. If the plot of U_{Li}/B *vs* b turns out to be linear, then the crack initiation resistance is constant regardless of different initial ligament length.

3. Determination of the steady state crack resistance (R_p)

The total energy consumed in ductile fracture includes blunting energy and the energy represented by the integration of the R-curve with respect to the cracking area, and may also include remote elastic energy loss and/or remote plastic energy loss. The blunting energy is the energy which is consumed in the vicinity of the crack tip prior to crack initiation. The total essential energy U_f for complete fracture throughout the ligament can be defined as the sum of the blunting energy and the integration of the R-curve with respect to the cracking area.

$$U_f = U_b + B\int_0^b R\,\mathrm{d}c \tag{3}$$

where U_b is the blunting energy, b is the initial ligament length, R is the resistance to crack propagation in terms of fracture energy as a function of c, and c is the amount of crack growth. The steady state crack resistance R_p can be determined utilizing this total essential energy U_f. An analysis about this has been reported [4] in detail, and a brief explanation will be given here. Graphical representation of the resistance to crack propagation as a function of c is the R-curve. The complete R-curve for ductile materials can be partitioned into ascending. plateau, and descending regions as shown in Fig. 2. The ascending region includes the resistance at crack initiation and an increasing resistance to crack growth due to a developing plastic deformation zone in the immediate vicinity of the crack tip. The plateau region occurs when the plastic deformation zone reaches and maintains its maximum size during crack growth. (In some special cases, the resistance may decrease after a maximum value due to re-sharpening of the crack tip or increasing crack propagation speed until it reaches a steady state value.) The descending region begins as the plastic deformation zone becomes confined by the specimen configuration and ends upon complete failure. Based on this R-curve, Equation 3 can be rewritten as

$$U_f = U_b + B\left[\int_0^{c1} R\,\mathrm{d}c + R_p(b - c1 - c2) + \int_{b-c2}^{b} R\,\mathrm{d}c\right] \tag{4}$$

where $c1$ and $c2$ are the base lengths of ascending and descending regions, and R_p is the steady state resistance value. As long as the initial ligament length b is greater than the sum of $c1$ and $c2$, the total essential energy U_f in Equation 4 is linearly dependent upon b since U_b and the integrations of the resistance R in the starting and the ending regions are constant regardless of the initial ligament length. Shown in Fig. 3 is a schematic plot of U_f/B with respect to initial ligament length b. When b is greater than the sum of $c1$ and $c2$, U_f can be written as

$$\frac{U_f}{B} = R_p\,b + \frac{U_{fo}}{B} \tag{5}$$

where U_{fo}/B is the intercept of the linearly extrapolated line down to zero ligament length. It can be shown using Equation 4 that $U_{fo}/B = U_b/B - V$ where V is the sum of the hatched areas in Fig. 2 which appear at the beginning and ending portion of the R-curve. Hence, when V is larger than U_b/B the intercept U_{fo}/B becomes negative. When the initial ligament length b is smaller than the sum of $c1$ and $c2$, U_f/B *vs* b plot should deviate from the linear line toward the origin since U_f must vanish when b approaches zero. The steady state resistance R_p can be determined from the slope of the linear portion of the U_f/B *vs* b plot.

4. Experimental

Three-point bend specimens were made from injection moulded bars of the Valox®, a PBT(polybutylene terephthalate) manufactured by the General Electric Company. The three-point bend test was chosen in order to induce slow stable crack propagation after the maximum load point. The width and thickness of the specimens were 12.7 and 3.2 mm respectively. The initial crack lengths varied from 0.635 mm to 11.43 mm in steps of 0.635 mm, which is from 0.05 to 0.90 in

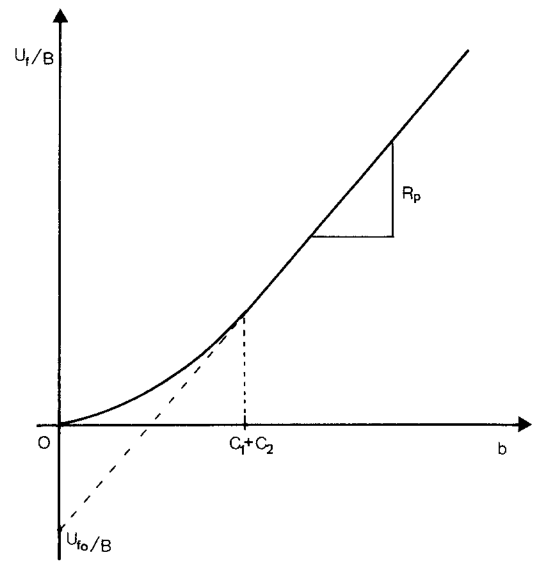

Figure 3 A schematic plot of U_f/B against initial ligament length b.

terms of a/W. The length of the initial crack included the length of the machined notch and the sharp crack created by pushing a razor blade into the tip of the machined notch. The width of the machined notch was 1.6 mm, and notch tip radius was 0.8 mm. The depth of the razor cut was approximately 0.5 mm.

One set of specimens was annealed after razor notching by placing the specimens in an oven at 145° C for 14 h at an absolute pressure of 3 mm Hg. These specimens were then allowed to cool in the oven for 10 h. Another set of specimens was not annealed in order to induce more plastic deformation during crack propagation.

Tests were performed at a cross head speed of 5 mm min^{-1} for the annealed set, and 20 mm min^{-1} for the unannealed set. The ambient temperature was between 23 and 25° C, and the relative humidity was about 42% for the annealed set and 58% for the unannealed set during the tests. The span in the three-point bend test was 51 mm for the annealed set, and 76 mm for the unnealed set. Load against load-point displacement graphs were recorded and the crack initiation points were marked on each loading curve. Crack initiations were observed with the aid of a microscope (Edmund Scientific Co., 50x). The areas under the curves were calculated numerically from the data points taken from the load-displacement record.

5. Result and discussion

Shown in Fig. 4 is the load *vs* load-point displacement record for annealed three-point bend specimens with $S/W = 4$. Solid dots denote the crack initiation points. Although the system becomes mechanically unstable when a/W is smaller than 0.26 [10] in three point bending with $S/W = 4$, none of the specimens showed fast fracture where the a/W ratios of the specimens represented by the highest four loading curves are smaller than 0.26. The term 'mechanically unstable' is used to represent a system which will show fast fracture if the material's crack resistance does not increase as the crack propagates. The highest four loading curves might have had different crack resistances during crack growth due to increased speed of crack propagation even though actual fast fracture did not occur. This will be discussed in detail later. The third and ninth curves from the top show relatively high energy consumption during extensive crack propagation compared to the others.

The total area under each loading curve was calculated based on the load *vs* load-point displacement record in Fig. 4. This area was taken as the total essential energy U_f for complete fracture assuming that remote plastic energy loss away from the crack was negligible. Shown in Fig. 5 is the plot of U_f/B as a function of b. The data from the third and ninth loading curves which were mentioned as having relatively high energy consumption are plotted in empty circles because of their different behaviour. The data from loading curves of mechanically unstable system are plotted in empty triangles. The filled circles show good linearity down to the smallest initial ligament size. The empty triangles are above the linear line indicating that the mechanically unstable specimens consumed more energy for complete fracture.

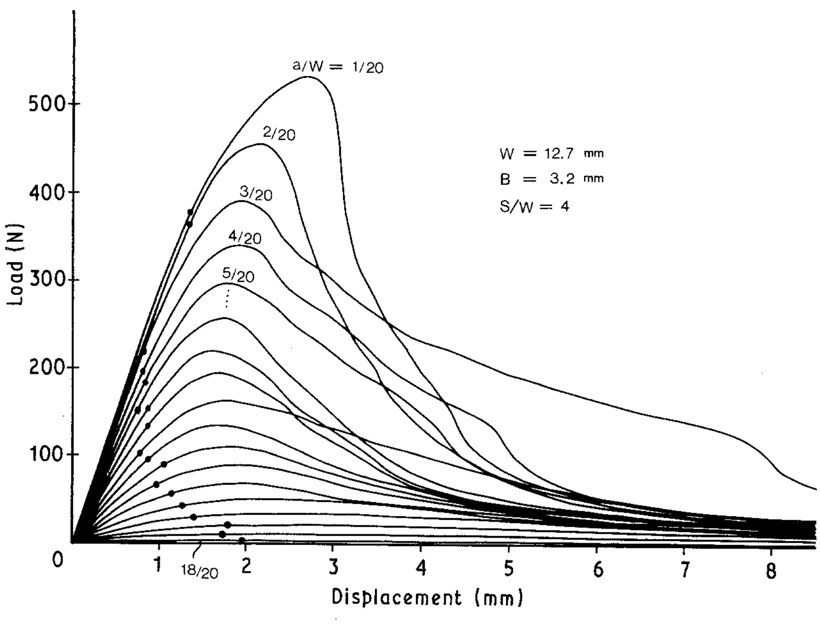

Figure 4 Load against load-point displacement record. (Three-point bending, PBT, annealed).

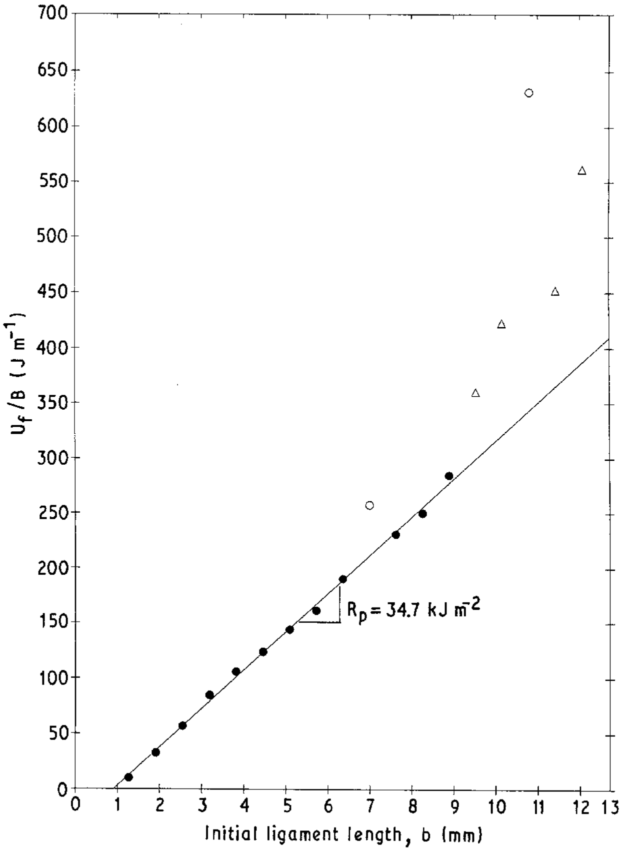

Figure 5 U_f/B against ligament length b, annealed.

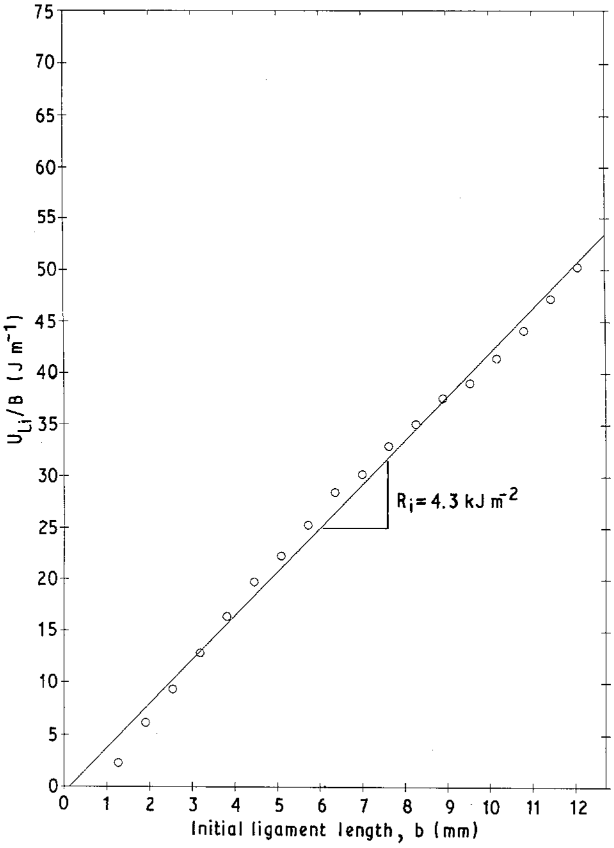

Figure 6 U_{Li}/B against initial ligament length b, annealed.

Attention should be paid when U_f is determined using a mechanically unstable loading configuration. For example, tensile testing without rotation is always mechanically unstable. The speed of crack propagation has a tendency to increase as the crack grows. Even though fast fracture does not always occur, the crack resistance may vary due to inconsistent crack propagation speeds. Hence, it is difficult to get compatible results from specimens of different initial crack lengths especially when the material's crack resistance is sensitive to the speed of crack propagation. Three-point bending is a mechanically stable loading configuration unless the initial crack size is smaller than a certain value which is dependent upon the S/W ratio. Therefore, reliable U_f values based on consistent slow stable crack growth can be achieved from three-point bend tests with enough initial crack length. This was the reason that three-point bend tests were used to determine U_f as a function of b in spite of the variations in the plastic deformation field near the end of the fracture process due to the compressive stress introduced by bending.

As was shown in Fig. 3, U_f/B is linear in b as long as the initial ligament length is larger than the sum of $c1$ and $c2$. Since the filled circles in Fig. 5 show good linearity down to the smallest initial ligament size, one can conclude that the sum of $c1$ and $c2$ in this case was sufficiently small. If the remote plastic energy loss had been significant, the plot of U_f *vs* b would not have shown the linear portion even if a proper range of b had been taken. The slope of the least squares linear fitted line to the filled circles was taken as R_p for the quasi-static steady crack propagation and the resulting value was 34.7 kJ m^{-2}.

Based on the load *vs* load-point displacement record in Fig. 4, U_{Li} which is the area enclosed by a loading curve, the locus line of crack initiation points, and the displacement axis was calculated for each loading curve. Shown in Fig. 6 is the plot of U_{Li} with respect to initial ligament length b. Since the plastic deformation zone around the crack tip prior to crack initiation is small enough not to be affected by the initial ligament length, the crack initiation resistance should not be a function of initial ligament length b in this case. Hence, the slope of the linear least squares fitted line can be taken as J_c for the given thickness. The resulting J_c value was 4.3 kJ m^{-2}.

Until now, results from annealed specimens have been shown. For these annealed specimens, the plastically deformed zone preceding the crack tip was so small that the end effects which occur when the plastic deformation around the crack tip is confined by the specimen geometry has not been seen in the experimental results. The same material without annealing, which was expected to show larger plastic deformation around the crack tip, was also tested. Shown in Fig. 7 is the load *vs* load-point displacement record of the unannealed three-point bend specimens. The ratio of span S to the specimen width W was 6 in this test. In three-point bending with $S/W = 6$, the system becomes mechanically unstable when a/W is smaller than 0.32 [10]. The loading curves of a/W ratio smaller than 0.32 are the top six curves in Fig. 7. Again, none of the specimens showed fast fracture.

The total area under each loading curve was calculated from the load against load-point displacement record shown in Fig. 7. Assuming again the remote plastic energy loss away from the crack was negligible,

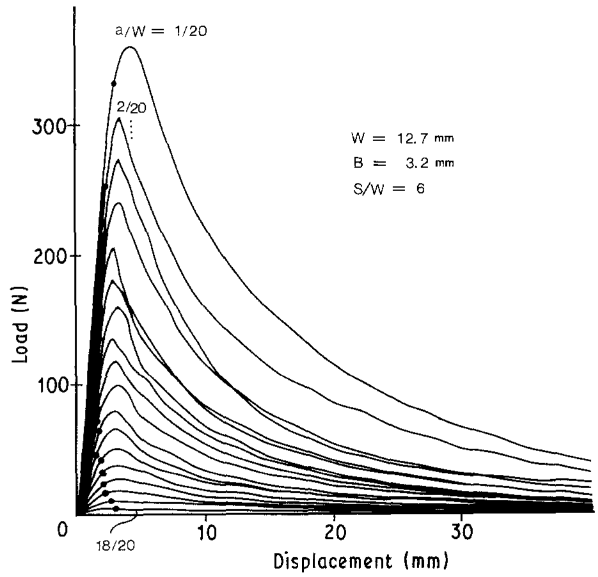

Figure 7 Load against load-point displacement record. (Three-point bending, PBT, unannealed).

this area was taken as the essential energy U_f for complete fracture. Shown in Fig. 8 is the plot of U_f/B as a function of b. The data from loading curves of mechanically unstable systems are plotted in empty triangles. The filled circles approach the origin non-linearly, but it is difficult to exactly divide this plot into different regions as shown in Fig. 3. Hence, it is difficult to evaluate exact R_p from Fig. 8 because the linear portion cannot be determined clearly. An approximated R_p value was determined from the plot, and the resulting value was about $120\,\mathrm{kJ\,m^{-2}}$.

Based on the load-displacement record in Fig. 7, U_{Li} was calculated along the line of crack initiation points for each loading curve. Shown in Fig. 9 is the plot of U_{Li}/B as a function of initial ligament length b. This plot shows a tendency to have larger slope as the initial ligament length b approaches the specimen width W. Based on Equation 2, the crack initiation resistance R_i (b) was calculated using a numerical differentiation with a first order forward difference. This crack initiation resistance R_i is plotted in Fig. 10. Since first order numerical differentiation has been employed, this plot shows large scatter. However, one can learn from this plot that T_i for this test decreases as the initial ligament length gets extemely small, and that R_i increases as the initial ligament length approaches the specimen width. This sudden increase of R_i at the very large initial ligament length can be attributed to the decreased stress concentration due to a large amount of plastic deformation around the small initial crack. On the other hand, the decreasing R_i at the extremely small ligament lengths can be attributed to the insufficient space around the crack tip for the plastic deformation. Excluding the first two and the last five data points, the crack initiation resistance R_i for reasonably deep cracks was determined using the least squares fitting to the U_{Li}/B *vs* b plot as shown in Fig. 9, and the resulting value was $5.0\,\mathrm{kJ\,m^{-2}}$. It should be noted that the crack initiation resistance at extremely small initial cracks was larger than that of the reasonably deep cracks.

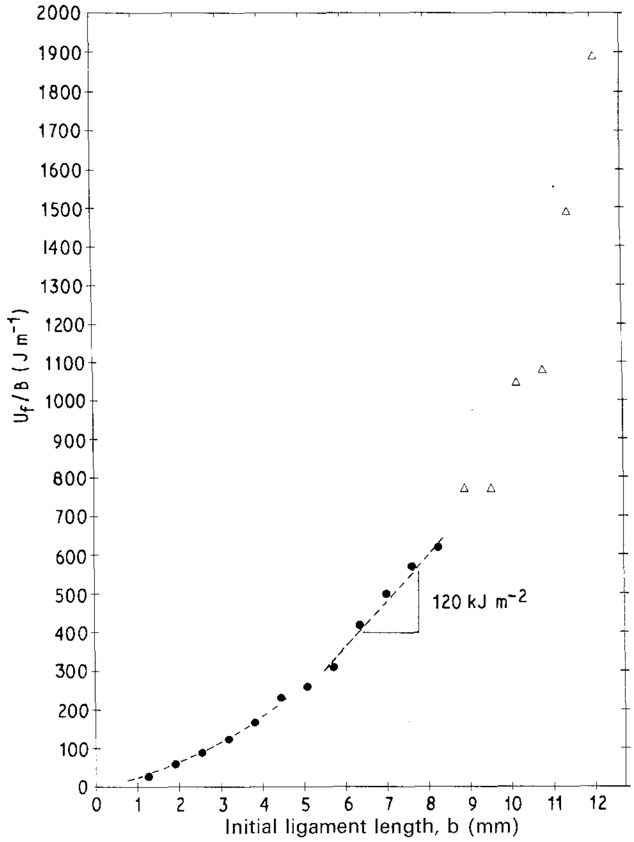

Figure 8 $U_f B$ against initial ligament b, unannealed.

6. Conclusion

The resistance to crack initiation and quasi-static crack propagation has been investigated for a polybutylene terephthalate (PBT). The crack initiation resistance was determined based on the generalized

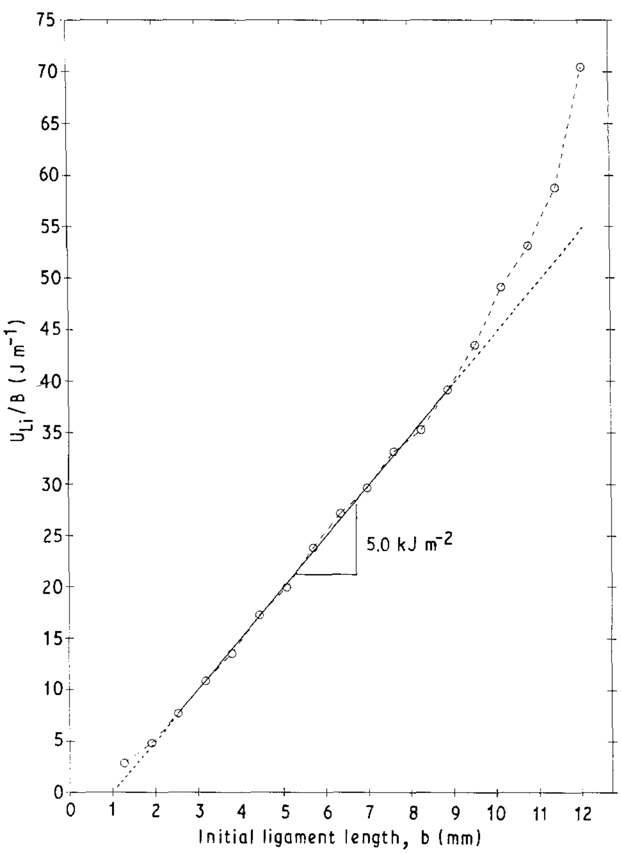

Figure 9 U_{Li}/B against initial ligament length b, unannealed.

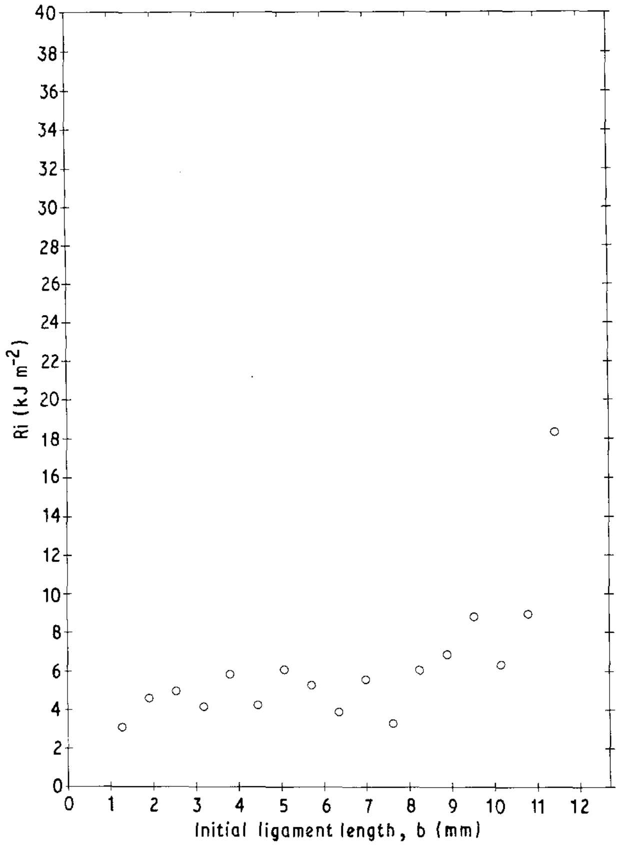

Figure 10 R_i (b) against initial ligament length b, unannealed.

locus method, and the steady state crack resistance R_p was determined utilizing the total essential energy U_f. Both annealed and unannealed specimens were tested using three-point bending. The unannealed specimens showed much larger plastic deformation around the crack tip than the annealed specimens. The crack initiation resistance R_i and the steady state crack resistance R_p were found to be 4.3 kJ m^{-2} and 34.7 kJ m^{-2} respectively for the annealed specimens. For the unannealed specimens, the crack initiation resistance R_i was found to be 5.0 kJ m^{-2} for reasonably deep cracks. The crack initiation resistances for extremely small initial cracks were higher because of reduced stress concentration due to a large amount of plastic deformation around the small initial cracks. The steady state crack resistance R_p for the unannealed specimens could not be clearly determined due to insufficient specimen size for the plastic deformation around the crack. The approximated R_p value was determined as 120 kJ m^{-2}.

Acknowledgement
The authors gratefully acknowledge the financial support provided by Massachusetts Center of Excellence Corporation. The authors thank General Electric Co. for supplying materials. Also, the authors wish to thank David M. Otterson for his assistance in preparing this manuscript.

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Received 2 December 1988
and accepted 31 May 1989